Task. If A is subset of real number and b is a real number, then show that

$$\inf(b+A) = b + \inf(A).$$

Proof. By definition

$$x = \inf(A)$$

if $x \leq a$ for every $a \in A$, and for every $\varepsilon > 0$ there exists $a \in A$ such that

$$a < x + \varepsilon.$$

Also notice that

$$b + A = \{b + a \mid a \in A\}.$$

It suffices to prove that

$$\inf(b+A) \geq b + \inf(A).$$

Then applying this identity to $A' = b + A$ and $b' = -b$ we will obtain that

$$\inf(b' + A') \geq b' + \inf(A'),$$

that is

$$\inf(-b + b + A) \geq -b + \inf(b + A)$$
$$\inf(A) \geq -b + \inf(b + A),$$
$$\inf(A) + b \geq \inf(b + A).$$

Which will imply that

$$\inf(A) + b = \inf(b + A).$$

Let $x = \inf(A)$ and $y = \inf(b + A)$. We should prove that

$$y \geq b + x.$$

Suppose $y < b + x$. This means that there exist $\varepsilon > 0$ such that

$$y < y + \varepsilon < b + x.$$

But then there exist $z \in b + A$ such that

$$z < y + \varepsilon < b + x.$$

Notice that z has the fowm $z = b + a$ for some $a \in A$, whence

$$b + a < b + x$$

and so

$$a < x$$

which contradicts to the assumption that $x = \inf(A) \leq a$.

Hence $y \geq b + x$. And so

$$\inf(A) + b = \inf(b + A).$$