

A random variable  $\xi$  is uniformly distributed over  $(-a, a)$ .

Moment generating function equals

$$M_{\xi}(t) = E(e^{t\xi}) = \int_{-\infty}^{\infty} e^{tx} f_{\xi}(x) dx = \int_{-a}^a \frac{e^{tx}}{2a} dx = \frac{1}{2a} \frac{e^{tx}}{t} \Big|_{x=-a}^{x=a} = \frac{1}{2at} (e^{ta} - e^{-ta})$$
$$E(\xi^{2n}) = M_{\xi}^{(2n)}(0)$$

For arbitrary  $a, b$ , random variable  $\eta$ , uniformly distributed on  $(c, d)$ , has such a moment:

$$m_k = \frac{1}{k+1} \sum_{i=0}^k c^i d^{k-i}$$

In our case  $k = 2n; c = -a; d = a$ ;

Thus,

$$E(\xi^{2n}) = \frac{1}{2n+1} \left( \sum_{i=0}^{2n} (-a)^i a^{2n-i} \right) = \frac{a^{2n}}{2n+1} \sum_{i=0}^{2n} (-1)^i = \frac{a^{2n}}{2n+1}$$