Reg and Jim are members of the avalanche rescue ski patrol. On a training exercise, they are 14 m apart and are searching for a beacon buried in the snow on a mountain. They both hone in on the signal that shows that they are on the same horizontal level as the beacon. Reg determines that the beacon is at an angle of 40 degrees from his line of sight to Jim. The angle from Jim's line of sight to Reg to where the beacon lies 55 degrees. Reg looks up the mountain to a rock that is sticking out of the snow directly above the beacon and records the angle of elevation to be 34 degrees. How deep is the beacon buried in the snow below the rock? Round to the nearest tenth of a meter.

Solution: Points, where the Reg, Jim and beacon are standing form a triangle. Let's name it $\Delta \mathrm{RBJ}$, where points R, J and B are corresponding to the positions of Reg, Jim and beacon.

From the condition of the problem we see, that $\angle \mathrm{BRJ} = 40^\circ$, $\angle \mathrm{RJB} = 55^\circ$, $\mathrm{RJ} = 14$ m.

It is known, that sum of the angles in the triangle is equal to 180°. Then, $\angle \mathrm{RBJ}$ is equal to:

$\angle \mathrm{RBJ} = 180^\circ - \angle \mathrm{BRJ} - \angle \mathrm{RJB} = 180^\circ - 40^\circ - 55^\circ = 85^\circ$.

From the law of sines, $\dfrac{\mathrm{RJ}}{\sin(\angle \mathrm{RBJ})} = \dfrac{\mathrm{RB}}{\sin(\angle \mathrm{RJB})}$, then $\mathrm{RB} = \mathrm{RJ} \cdot \dfrac{\sin(\angle \mathrm{RJB})}{\sin(\angle \mathrm{RBJ})} = 14 \cdot \dfrac{\sin 55^\circ}{\sin 85^\circ} = 11.5$ m.

Now let's consider the triangle $\Delta \mathrm{RBO}$, where O is the position of the rock above the beacon. It's the right triangle with legs RB and BO. From the condition of the problem we see, that $\angle \mathrm{BRO} = 34^\circ$.

According to the definition of tangent, $\tan(\angle \mathrm{BRO}) = \dfrac{\mathrm{BO}}{\mathrm{RB}}$. Then, desired height of the rock above the beacon BO can be calculated as: $\mathrm{BO} = \mathrm{RB} \cdot \tan(\angle \mathrm{BRO}) = 11.5 \cdot \tan 34^\circ \approx 7.8$ m.

Answer: 7.8 m.