

Take the integral:

$$\int \sqrt{9 + \frac{4}{x^{\frac{2}{3}}}} dx$$

For the integrand $\sqrt{9 + \frac{4}{x^{\frac{2}{3}}}}$, substitute $u = x^{-\frac{2}{3}}$ and $du = -\frac{2}{3x^{\frac{5}{3}}} dx$

$$\int \sqrt{9 + \frac{4}{x^{\frac{2}{3}}}} dx = -\frac{3}{2} \int \frac{\sqrt{4u+9}}{u^{\frac{5}{2}}} du$$

For the integrand $\frac{\sqrt{4u+9}}{u^{\frac{5}{2}}}$, substitute $s = \sqrt{u}$ and $ds = \frac{1}{2\sqrt{u}} du$

$$\text{integral} = -3 \int \frac{\sqrt{4s^2+9}}{s^4} ds$$

For the integrand $\frac{\sqrt{4s^2+9}}{s^4}$, substitute $s = \frac{3\tan(p)}{2}$ and $ds = \frac{3}{2}\sec^2(p)dp$.

Then $\sqrt{4s^2+9} = \sqrt{9\tan^2(p)+9} = 3\sec(p)$ and $p = \tan^{-1}(\frac{2s}{3})$

$$\text{integral} = -\frac{27}{2} \int \frac{16}{81} \cot(p) \csc^3(p) dp = -\frac{8}{3} \int \cot(p) \csc^3(p) dp$$

substitute $w = \csc(p)$ and get

$$\begin{aligned} \text{integral} &= -\frac{8}{3} \int w^2 dw = \frac{8 \csc^3(p)}{9} + \text{constant} = \frac{(4s^2+9)^{\frac{3}{2}}}{9s^3} + \text{constant} \\ &= \frac{1}{9} x \left(\frac{4}{x^{\frac{2}{3}}} + 9 \right)^{\frac{3}{2}} + \text{constant} \end{aligned}$$