Task. Determine the limit:

$$\lim_{x\to 0}\frac{4e^x-2e^{-x}-2}{x}$$

Solution. We can use L'Hôpital's rule claiming that if f and g are two differentiable functions at $x=0$ then $f(0)=g(0)=0$, then

$$\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{f'(x)}{g'(x)}.$$

Let $f(x)=4e^x-2e^{-x}-2$, and $g(x)=x$. Then

$$f(0)=4-2-2=0, \qquad g(0)=0,$$

whence

$$\begin{aligned}\lim_{x\to 0}\frac{4e^x-2e^{-x}-2}{x}&=\lim_{x\to 0}\frac{f(x)}{g(x)}=\lim_{x\to 0}\frac{f'(x)}{g'(x)}=\lim_{x\to 0}\frac{(4e^x-2e^{-x}-2)'}{x'}\\&=\lim_{x\to 0}\frac{4e^x+2e^{-x}}{1}=\lim_{x\to 0}(4e^x+2e^{-x})=4+2=6.\end{aligned}$$