

Find the equation of a locus of point whose distance from the point $(2, -2)$ is equal to its distance from the line $x - y = 0$

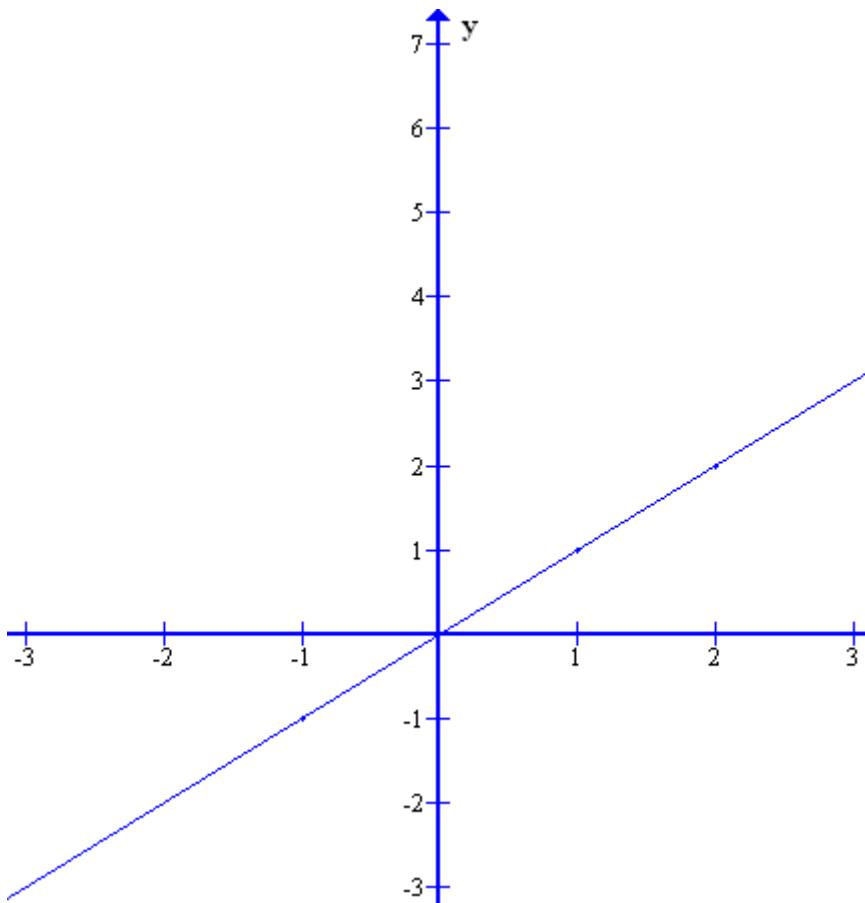
Solution:

The equation to a locus is the condition which the coordinates of each of the points of that locus and only those points must satisfy. In other words, the equation to a locus is nothing but the geometrical property expressed in algebraic language. Conversely, let (x, y) represent any point on the locus, x and y satisfy the equation of the locus and every point satisfying the equation lies on the locus. If the functional relation between x, y be $f(x, y) = 0$, then we say $f(x, y) = 0$ is the Cartesian equation of the locus. Thus in coordinate geometry a locus is represented by an equation. The set of points (x, y) satisfying a given equation $f(x, y) = 0$ is called the locus or graph of the equation.

In our case we have to find the equation for locus of a point $(2, -2)$ is equal to its distance from the line $x - y = 0$ (or we can write $x = y$)

Let $P(x, y)$ be a point satisfying the geometrical condition, call point $A(2, -2)$ and $O(0, 0)$

We can graph the line $x - y = 0$



$$PA = OP$$

$$\Rightarrow PA^2 = OP^2$$

So we can find from equation of distance between points:

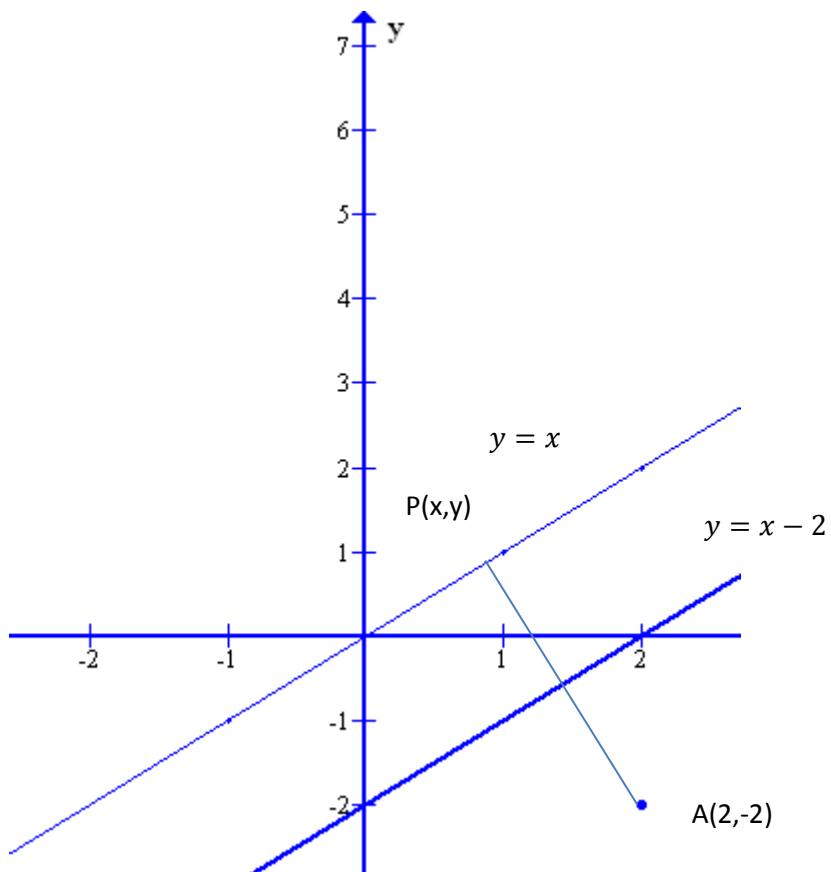
$$(x - 2)^2 + (y + 2)^2 = x^2 + y^2$$

$$x^2 - 4x + 4 + y^2 + 4y + 4 = x^2 + y^2$$

$$-4x + 4 + 4y + 4 = 0$$

$$4y + 8 = 4x$$

$$y = x - 2$$



Answer: $y = x - 2$