

**Task.** Determine the limit:

$$\lim_{x \rightarrow \infty} \left[ \frac{x}{1+x} \right]^x.$$

**Solution.** We can use the following limit:

$$\lim_{x \rightarrow \infty} \left[ 1 + \frac{1}{x} \right]^x = e.$$

In our case

$$\lim_{x \rightarrow \infty} \left[ \frac{x}{1+x} \right]^x = \lim_{x \rightarrow \infty} \left[ \frac{1+x}{x} \right]^{-x} = \lim_{x \rightarrow \infty} \left[ 1 + \frac{1}{x} \right]^{-x} = \lim_{x \rightarrow \infty} \left( \left[ 1 + \frac{1}{x} \right]^x \right)^{-1} = e^{-1}.$$