

How to change biquadratic polynomial into perfect square?

Solution:

Let our polynomial  $f(x) = ax^2 + bx + c$ . Rewrite it into perfect square:

$$\begin{aligned} f(x) &= ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right) = a\left(\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right) = \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \text{ and } \left(x + \frac{b}{2a}\right) \text{ is our perfect square.} \end{aligned}$$