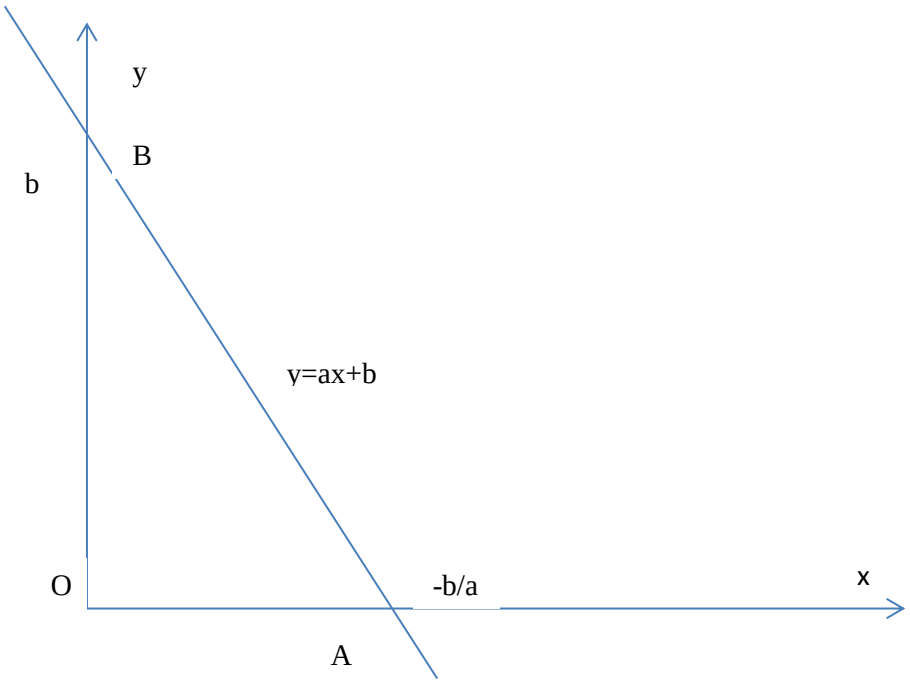
A line segment has one endpoint A on the x axis and the other endpoint B, on the y-axis. It passes through the point (1, 2). If O is the point (0, 0), for what values of angle OAB is the area of triangle AOB a minimum? What is the minimum area of triangle AOB?

Equation of the line: $y = ax + b$. If line passes through the point (1, 2):

$2 = 1 * a + b \quad => \quad a = 2 - b$



Y-intercept equals (OB): $y(0) = a * 0 + b = b$

X-intercept equals (AO): $y = 0 = ax + b \quad => \quad x = -b/a$

So, area of triangle AOB equals:

$S = \frac{1}{2}\, b * \left(-\frac{b}{a}\right)$

If $a = 2 - b$:

$S = \frac{1}{2}\frac{b^2}{b-2}$

Minimum value if $\frac{dS}{db} = 0$

$\frac{dS}{db} = \frac{1}{2}\frac{2b(b-2)-b^2}{(b-2)^2} = \frac{1}{2}\, b\, \frac{b-4}{(b-2)^2}$

$b - 4 = 0$

$b = 4$

$\tan(OAB) = \frac{OB}{OA} = \frac{b}{-\frac{b}{a}} = -a$

$a = 2 - b = 2 - 4 = -2$

$OAB = \arctan(2) = 63.4$

Area equals:

$S = \frac{1}{2}\ b * \left(-\frac{b}{a}\right) = \frac{1}{2}\frac{4^2}{2} = 4$

Answer: $AB = 63.4$, $S = 4$