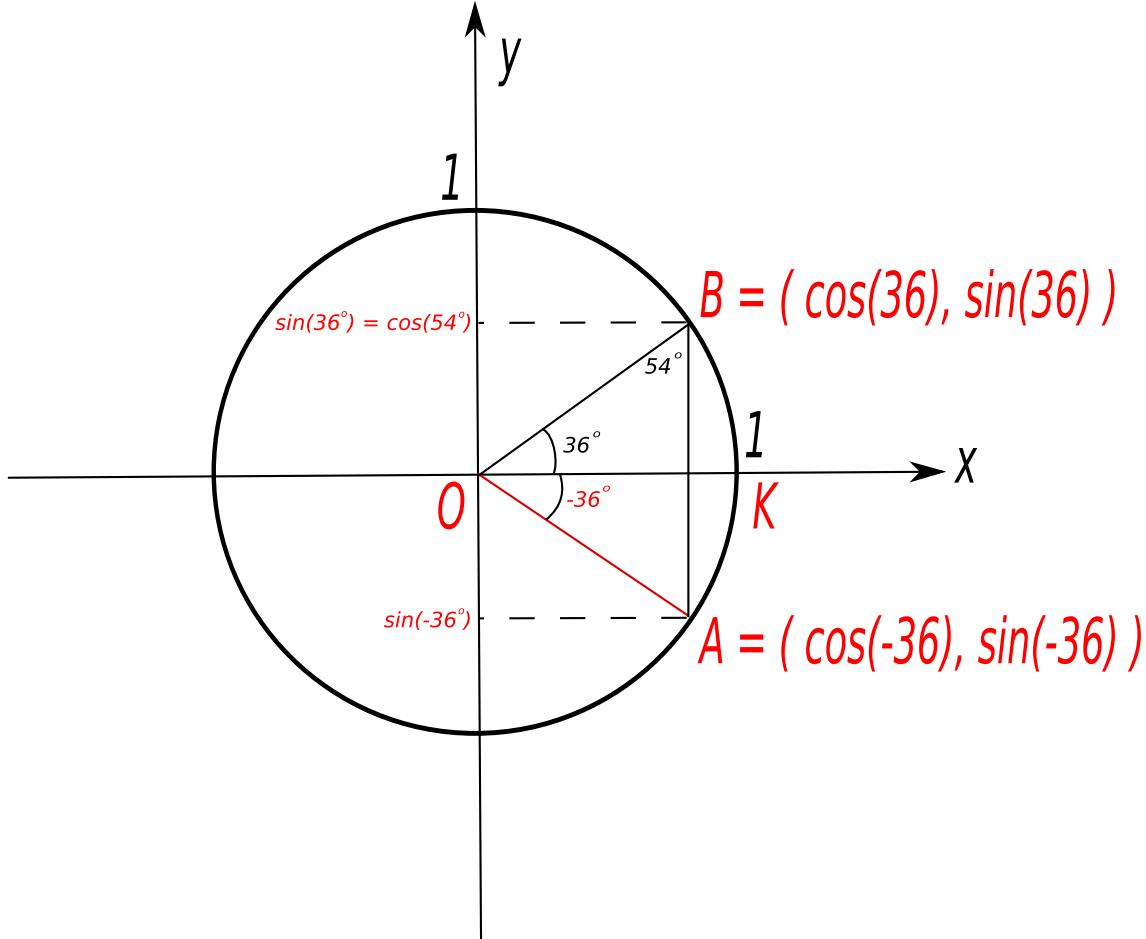


Task. Express $\sin(-36^\circ)$ in terms of

- the sin of an acute angle and
- the cos of an acute angle.

Solution. Consider the unit circle in the plane (x, y) :



Let A be the point on this circle in the lower half-plane such that $\angle AOK = 36^\circ$. Then $(\cos(-36), \sin(-36))$ are, by definition, the coordinates of A . Hence

$$\sin(-36) = -AK.$$

We should express the second coordinate of A via sin and cos of acute angle.

Consider the symmetric point B on the circle, laying in the upper half-plane and such that $\angle BOK = 36^\circ$. Then $B = (\cos(36^\circ), \sin(36^\circ))$, and in particular, $BK = \sin(36^\circ)$. It is also evident, that the triangles OBK and OAK are equal, so $BK = AK$, and therefore

$$\sin(-36^\circ) = -AK = -BK = -\sin(36^\circ).$$

Furthermore,

$$\sin 36^\circ = \sin \angle BOK = \frac{BK}{OB} = \cos KBO = \cos 54^\circ.$$

Answer. $\sin(-36^\circ) = -\sin 36^\circ = \cos 54^\circ$.