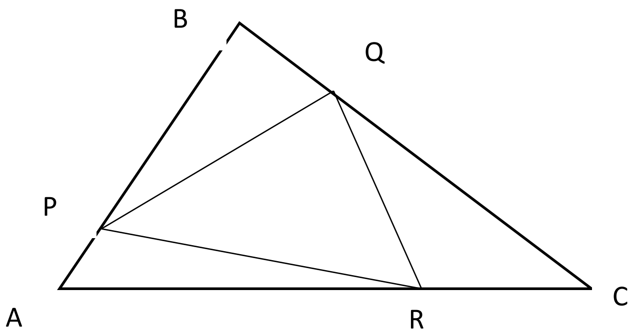
P, Q, R are the three points of a ΔABC such that AP:PB =2:5 and BQ:QC=2:5 and CR:RA =2:5. Find the ratio of the area of the ΔABC and ΔPQR.

Solution.



$\frac{AP}{PB}=\frac{2}{5}, PB=\frac{5AP}{2}, AB=AP+PB=AP+\frac{5AP}{2}=\frac{7AP}{2}, AP=\frac{2AB}{7}, PB=\frac{5AB}{7}.$

$\frac{BQ}{QC}=\frac{2}{5}, QC=\frac{5BQ}{2}, BC=BQ+QC=\frac{7BQ}{2}, BQ=\frac{2BC}{7}, QC=\frac{5BC}{7}.$

$\frac{CR}{RA}=\frac{2}{5}$, so $CR=\frac{2AC}{7}, RA=\frac{5AC}{7}.$

Let’s consider ΔAPR:

$S_{\Delta APR}=\frac{1}{2}AP\cdot RA\sin\angle A, S_{\Delta ABC}=\frac{1}{2}AB\cdot AC\sin\angle A$

$$\frac{S_{\Delta APR}}{S_{\Delta ABC}}=\frac{\frac{1}{2}AP\cdot RA\sin\angle A}{\frac{1}{2}AB\cdot AC\sin\angle A}=\frac{\frac{1}{2}\cdot\frac{2}{7}AB\cdot\frac{5}{7}AC\sin\angle A}{\frac{1}{2}AB\cdot AC\sin\angle A}=\frac{10}{49}$$

Let consider ΔPBQ:

$S_{\Delta PBQ}=\frac{1}{2}BQ\cdot PB\sin\angle B, S_{\Delta ABC}=\frac{1}{2}AB\cdot BC\sin\angle B$

$$\frac{S_{\Delta PBQ}}{S_{\Delta ABC}}=\frac{\frac{1}{2}BQ\cdot PB\sin\angle B}{\frac{1}{2}AB\cdot BC\sin\angle B}=\frac{10}{49}$$

Let consider ΔQCR:

$S_{\Delta QCR}=\frac{1}{2}QC\cdot CR\sin\angle C, S_{\Delta ABC}=\frac{1}{2}BC\cdot AC\sin\angle C$

$$\frac{S_{\Delta QCR}}{S_{\Delta ABC}}=\frac{\frac{1}{2}QC\cdot CR\sin\angle C}{\frac{1}{2}BC\cdot AC\sin\angle C}=\frac{10}{49}$$

Let’s consider ΔPQR:

$S_{\Delta PQR}=S_{\Delta ABC}-S_{\Delta APR}-S_{\Delta PBQ}-S_{\Delta QCR}=S_{\Delta ABC}-\frac{10}{49}S_{\Delta ABC}-\frac{10}{49}S_{\Delta ABC}-\frac{10}{49}S_{\Delta ABC}=\frac{19}{49}S_{\Delta ABC}.$

$$\frac{S_{\Delta ABC}}{S_{\Delta PQR}}=\frac{S_{\Delta ABC}}{\frac{19}{49}S_{\Delta ABC}}=\frac{49}{19}$$

Answer.

$$\frac{S_{\Delta ABC}}{S_{\Delta PQR}} = \frac{49}{19}$$