Find the equation of a locus of moving point such that the slope of line joining the point to $A(1,3)$ is three times that of the slope of the line joining the point to $B(3,1)$

Solution:

A locus describes a set of points $P(x, y)$ that obeys certain conditions, or a single point $P(x, y)$ that moves along a certain path.

If a point moves on a plane satisfying some given geometrical condition then the path trace out by the point in the plane is called its locus. By definition, a locus is determined if some geometrical condition are given. Evidently, the co-ordinate of all points on the locus will satisfy the given geometrical condition. The algebraic form of the given geometrical condition which is satisfied by the co-ordinate of all points on the locus is called the equation to the locus of the moving point. Thus, the co-ordinates of all points on the locus satisfy its equation of locus: but the co-ordinates of a point which does not lie on the locus do not satisfy the equation of locus. Conversely, the points whose co-ordinates satisfy the equation of locus lie on the locus of the moving point.

Let the moving point be (x, y). Slope of moving point with respect to point $A(1,3) = \frac{(y-3)}{(x-1)}$. Slope of the moving point with respect to point $B(3,1) = \frac{(y-1)}{(x-3)}$.

We are given that the slope of the line joining (x, y) with point $A(1,3)$ is three times that of the slope of the line joining the point to $B(3,1)$ therefore our equation becomes:

$$\frac{(y-3)}{(x-1)} = \frac{3(y-1)}{(x-3)}$$

Simplify our equation: $3(y-1)(x-1) = (y-3)(x-3)$

$$3(xy - y - x + 1) = (xy - 3y - 3x + 9)$$

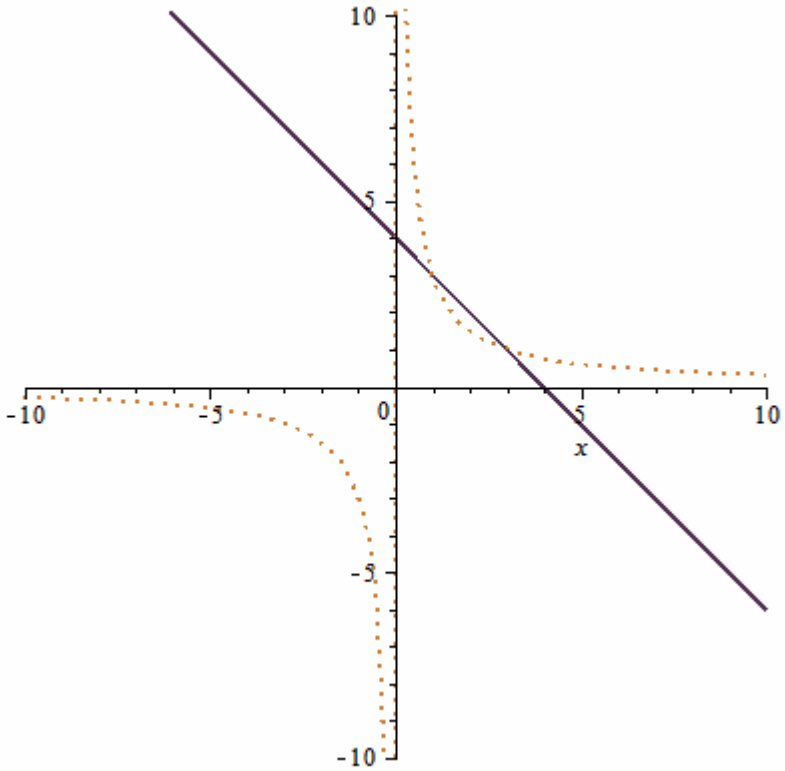
$$3xy - 3x - 3y + 3 = xy - 3y - 3x + 9$$

$$3xy - 3x - 3y + 3 - xy + 3x + 3y - 9 = 0$$

$$2xy - 6 = 0$$

So, we find the equation of a locus of moving point: $y = \frac{3}{x}$

The equation of a locus of moving point can be represented graphically.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