Why is $tan\ \theta$ = slope of line?

Answer:

To explain why is $tan\ \theta$ = slope of line we need to define the slope. The slope of a line is a number that measures its "steepness", usually denoted by the letter m. It is the change in y for a unit change in x along the line. The slope of a line (also called the gradient of a line) is a number that describes how "steep" it is. Also we know one of the most important properties of a straight line is in how it angles away from the horizontal. This concept is reflected in something called the "slope" of the line.

Consider a straight line that passes through the points **A**$(\boldsymbol{x_1 y_1})$ and **B**$(\boldsymbol{x_2 y_2})$**.** We know that the gradient, m, of the line is defined by:

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{BC}{AC} = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_2 \neq x_1 (From\ the\ graph\ below)$$

Where, m = slope of the line.

Let θ be the angle (inclination angle). Then in the right angled triangle, Perpendicular = $y_2 - y_1$and base =$x_2 - x_1$

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{BC}{AC} (\frac{Perpendicular}{\text{Base}})$$

Therefore, $\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

From the definition of the line and $\tan\theta$ we get that $slope\ of\ the\ line\ =\ tan\ \theta$

$$m = tan\ \theta \ \text{ slope of the line}$$