Task. What is the vertex, axis of symmetry, maximum or minimum value, and domain and range of the function $f(x) = 3(x+6)^2 + 1$?

Solution. 1) This function is defined for all x, and so its domain is $(-\infty, +\infty)$.

2) Notice that

$$f(x) = 3(x+6)^2 + 1 = 3(x^2 + 12x + 36) + 1 = 3x^2 + 36x + 109.$$

So it is given by quadratic equation, and therefore its graph is a parabola. Hence it has a vertical axis of symmetry passing through its vertex. Also it has no other axes of symmetry.

Moreover, since the coefficient at $(x+6)^2$ is $3 > 0$, the branches of this parabola are directed upward, and so f is not bounded from the above. In other words, it does not achive maximal value.

3) The vertex of this parbola, i.e. a unique critical point, is at $x = -6$. Indeed, the critical points are the solutions of the equation $f'(x) = 0$. In our case we have that

$$f'(x) = (3x^2 + 36x + 109)' = 6x^2 + 36 = 0$$

whence

$$x = \frac{-36}{6} = -6.$$

Therefore, the axis of symmetry of the graph of f is the line given by the formula

$$x = -6.$$

4) The value of f at its vertex is

$$f(-6) = 3(-6+6)^2 + 1 = 1.$$

This is the minimal value of f.

Since f is continuous and unbounded from above, its range is $[1, +\infty)$.

Answer.

- vertex: $x = -6$
- axis of symmetry: the vertical line $x = 6$
- maximum value: does not exist
- minimum value: $f(-6) = 1$
- domain: $(-\infty, +\infty)$,
- range: $[1, +\infty)$.