Question #29898

Given that sin(theta)=15/17 and theta is in quadrant 2, determine sin(2theta), cos(2theta), and tan(2theta) Which quadrant is 2theta in?

Solution.

If θ is in quadrant 2, then $\cos(\theta) \leq 0$ and $\cos(\theta) = -\sqrt{1 - sin^2(\theta)} = -\sqrt{1 - \frac{225}{289}} = -\frac{8}{17}$.

Since $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, then

$$\sin(2\theta) = 2 \cdot \frac{15}{17} \cdot \left(-\frac{8}{17}\right) = -\frac{240}{289}.$$

Using the formula $\cos(2\theta) = 2cos^2(\theta) - 1$, we obtain

$$\cos(2\theta) = 2\frac{64}{289} - 1 = -\frac{161}{289}.$$

Then $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{240}{161}$.

Taking into account that $\cos(2\theta) < 0$ and $\sin(2\theta) < 0$, we conclude that 2θ is in quadrant 3.

Answer. $\sin(2\theta) = -\frac{240}{289}, \cos(2\theta) = -\frac{161}{289}, \tan(2\theta) = \frac{240}{161}$.