Question #29875

If F'(x) = f(x) then the antiderivative for 1 to 3 of f(2x)dx =

Solution. To find the antiderivative for a function $f(2x)$, we need to integrate it $\int_1^3 f(2x)dx$.

Since $F'(x) = f(x)$, then $\int_1^3 f(x)dx = F(x)|_1^3$. To evaluate $\int_1^3 f(2x)dx$, we make a substitution $t = 2x$. Thus, $x = \frac{t}{2}$ and $dx = d\left(\frac{t}{2}\right) = \frac{1}{2}dt$. Since $1 \le x \le 3$, then $2 \le t \le 6$.

It follows that

$\int_1^3 f(2x)dx = \int_2^6 \frac{1}{2}f(t)d(t) = \frac{1}{2} F(t)|_2^6 = \frac{1}{2}\big(F(6) - F(2)\big)$.

Answer. $\frac{1}{2}\big(F(6) - F(2)\big)$.