Task. Three boys agree to meet at midday tomorrow. The probability of each boy remembering to meet is 3/4, 4/5, 5/8. Calculate the probability that at least two of the boys will remember to meet.

Solution. Consider the following 4 events:
A_{011} - "only 1st boy will forget to meet"
A_{101} - "only 2dn boy will forget to meet"
A_{110} - "only 3rd boy will forget to meet"
A_{111} - "only boys will remember to meet"

Thus 0 (resp 1) at poisition i means that i-th boy will forget (resp. remember) to meet.

Then these events are collectively exhaustive.

We should compute the probability p that at least two of the boys will remember to meet, i.e. the probability

$$p = P(A_{111} \cup A_{110} \cup A_{101} \cup A_{011}).$$

Since the events are collectively exhaustive it follows that

$$p = P(A_{111} \cup A_{110} \cup A_{101} \cup A_{011}) = P(A_{111}) + P(A_{110}) + P(A_{101}) + P(A_{011}).$$

Let A_i be the event that the i-th boy will remember to meet, so

$$P(A_1) = 3/4, \qquad P(A_2) = 4/5, \qquad P(A_3) = 5/8.$$

Then

$$\begin{aligned} A_{111} &= A_1 \cap A_2 \cap A_3, \\ A_{110} &= A_1 \cap A_2 \cap \overline{A_3}, \\ A_{101} &= A_1 \cap \overline{A_2} \cap A_3, \\ A_{011} &= \overline{A_1} \cap A_2 \cap A_3. \end{aligned}$$

Assume that events A_1, A_2 and A_3 are independent. This means that

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3) &= P(A_1) * P(A_2) * P(A_3) = \frac{3}{4} * \frac{4}{5} * \frac{5}{8} = \frac{60}{160}, \\ P(A_1 \cap A_2 \cap \overline{A_3}) &= P(A_1) * P(A_2) * (1 - P(A_3)) = \frac{3}{4} * \frac{4}{5} * \frac{3}{8} = \frac{36}{160} \\ P(A_1 \cap \overline{A_2} \cap A_3) &= P(A_1) * (1 - P(A_2)) * P(A_3) = \frac{3}{4} * \frac{1}{5} * \frac{5}{8} = \frac{15}{160} \\ P(\overline{A_1} \cap A_2 \cap A_3) &= (1 - P(A_1)) * P(A_2) * P(A_3) = \frac{1}{4} * \frac{4}{5} * \frac{5}{8} = \frac{20}{160} \end{aligned}$$

Hence the probability p that at least two of the boys will remember to meet is equal to

$$\begin{aligned} p &= P(A_{111}) + P(A_{110}) + P(A_{101}) + P(A_{011}) = \frac{60}{160} + \frac{36}{160} + \frac{15}{160} + \frac{20}{160} \\ &= \frac{60 + 36 + 15 + 20}{160} = \frac{131}{160}. \end{aligned}$$

Answer. $p = \frac{131}{160}$.