

Show that there are at least three distinct points $x_1, x_2,$ and x_3 such that $f(x_1)=f(x_2)=f(x_3)=10$, where $f(x)=x^3/(x^2-1)$

$$\frac{x^3}{x^2-1}=10$$

The binomial can be factored using the difference of squares formula, because both terms are perfect squares. The difference of squares formula is $a^2 - b^2 = (a - b)(a + b)$.

$$\frac{x^3}{(x-1)(x+1)}=10$$

Since the variable is in the denominator on the left-hand side of the equation, this can be solved as a ratio. For example, $\frac{A}{B} = C$ is equivalent to

$$\frac{A}{C} = B.$$

$$\frac{x^3}{10} = (x-1)(x+1)$$

Since x is on the right-hand side of the equation, switch the sides so it is on the left-hand side of the equation.

$$(x-1)(x+1) = \frac{x^3}{10}$$

Check that all solutions found are valid and are part of the domain by substituting them into the original equation.

$$x = \frac{x^3}{10}$$

Multiply each term in the equation by 10.

$$(x-1)(x+1) \cdot 10 = \frac{x^3}{10} \cdot 10$$

Multiply $(x-1)(x+1)$ by 10 to get $10(x-1)(x+1)$.

$$10(x-1)(x+1) = \frac{x^3}{10} \cdot 10$$

Simplify the right-hand side of the equation by simplifying each term.

$$10(x-1)(x+1) = x^3$$

Multiply each term in the first group by each term in the second group using the FOIL method. FOIL stands for First Outer Inner Last, and is a method of multiplying two binomials. First, multiply the first two terms in each binomial group. Next, multiply the outer terms in each group, followed by the inner terms. Finally, multiply the last two terms in each group.

$$10(x \cdot x + x \cdot 1 - 1 \cdot x - 1 \cdot 1) = x^3$$

Simplify the FOIL expression by multiplying and combining all like terms.

$$10(x^2 - 1) = x^3$$

Multiply 10 by each term inside the parentheses.

$$10x^2 - 10 = x^3$$

Since x^3 contains the variable to solve for, move it to the left-hand side of the equation by subtracting x^3 from both sides.

$$10x^2 - 10 - x^3 = 0$$

Move all terms not containing x to the right-hand side of the equation.

$$-x^3 + 10x^2 - 10 = 0$$

Divide each term in the equation by -1 .

$$x^3 - 10x^2 + 10 = 0$$

The solutions of the polynomial equation were found with the Durand-Kerner Method. There are 0 imaginary solutions.

$$x = 1.0575, -0.9554, 9.8979$$