

prove : $\tan(x) + \sec(x) = \cos(x) / 1 - \sin(x)$

By definition, the tangent (tan) of an angle is the ratio of the sine to the cosine:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

and the reciprocal function secant (sec) is the reciprocal of the cosine:

$$\sec(x) = \frac{1}{\cos(x)}$$

Therefore:

$$\tan(x) + \sec(x) = \frac{\sin(x)}{\cos(x)} + \frac{1}{\cos(x)} = \frac{\sin(x)+1}{\cos(x)}$$

$$\text{multiply by } 1 = \frac{1-\sin(x)}{1-\sin(x)}:$$

$$\frac{(\sin(x)+1)(1-\sin(x))}{\cos(x)(1-\sin(x))} = \frac{1-\sin^2(x)}{\cos(x)(1-\sin(x))}$$

The basic relationship between the sine and the cosine is the Pythagorean trigonometric identity:

$$\cos^2(x) + \sin^2(x) = 1 \quad \Rightarrow \quad 1 - \sin^2(x) = \cos^2(x)$$

Therefore:

$$\tan(x) + \sec(x) = \frac{\cos^2(x)}{\cos(x)(1-\sin(x))} = \frac{\cos(x)}{1-\sin(x)}$$