1) Solve the equation:

$$sin^2\frac{x}{2} - 2 = 0$$

Solution:

$$sin^2\frac{x}{2} - 2 = 0$$

Add 2 to both sides

$$sin^2\frac{x}{2} = 2$$

Take the square root of both sides

$$\sin\frac{x}{2} = \sqrt{2} \quad \text{or} \quad \sin\frac{x}{2} = -\sqrt{2}$$

Look at the first equation $\sin\frac{x}{2} = \sqrt{2}$.

Take the inverse sine of both sides:

$$\frac{x}{2} = \pi - \arcsin\sqrt{2} + 2\pi n_1 \quad n_1 \in Z$$

or

$$\frac{x}{2} = \arcsin\sqrt{2} \ + 2\pi n_2 \ \ n_2 \in Z$$

Multiply both sides by 2:

$$x = 2\pi - 2\arcsin\sqrt{2} + 2\pi n_1 \ \ n_1 \in Z$$

$$x = 2\arcsin\sqrt{2} + 4\pi n_2 \ \ n_2 \in Z$$

Look at the second equation $\sin\frac{x}{2} = -\sqrt{2}$

Take the inverse sine of both sides:

$$\frac{x}{2} = \pi + \arcsin\sqrt{2} + 2\pi n_3 \ \ n_3 \in Z$$

$$\frac{x}{2} = 2\pi n_4 - \arcsin\sqrt{2} \ \ n_4 \in Z$$

Multiply both sides by 2:

$$x = 2\pi + 2\arcsin\sqrt{2} + 4\pi n_3 \ \ n_3 \in Z$$

$$x = 4\pi n_4 - 2\arcsin\sqrt{2} \ \ n_4 \in Z$$

Answer:

$$x = 2\pi - 2\arcsin\sqrt{2} + 2\pi n_1 \ \ n_1 \in Z$$

$$x = 2\arcsin\sqrt{2} + 4\pi n_2 \ \ n_2 \in Z$$

$$x = 2\pi + 2\arcsin\sqrt{2} + 4\pi n_3 \ \ n_3 \in Z$$

$$x = 4\pi n_4 - 2\arcsin\sqrt{2} \ \ n_4 \in Z$$

2) Solve for exact solutions over [0,2pie) interval:

$$sin^2\left(\frac{x}{2} - 2\right) = 0$$

Solution:

Take the square root of both sides:

$$\sin\left(\frac{x}{2} - 2\right) = 0$$

Take the inverse sine of both sides:

$$\frac{x}{2} - 2 = \pi n \ \ n \in Z$$
$$0 \le x < 2\pi$$

so

$$\frac{x}{2} - 2 = 0$$
$$\frac{x}{2} - 2 = \pi$$

Add 2 to both sides:

$$\frac{x}{2} = 2$$
$$\frac{x}{2} = \pi + 2$$

Multiply both sides by 2:

$$x = 4$$

$$x = 2\pi + 4$$

Answer:

$$x = 4$$

$$x = 2\pi + 4$$