If $sinB = 3sin(2A + B)$ prove that $2tanA + tan(A + B) = 0$

Proof:

$2\tan A + \tan(A + B) = 0$

$\text{LHS} = 2\tan A + \tan(A + B) \quad (1)$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)}$$

Substituting into (2):

$$\text{LHS} = 2\frac{\sin A}{\cos A} + \frac{\sin(A + B)}{\cos(A + B)} = \frac{2\sin A\cos(A + B) + \sin(A + B)\cos A}{\cos(A + B)\cos A}$$

$$\text{LHS} = \frac{\sin A\cos(A + B) + \sin A\cos(A + B) + \sin(A + B)\cos A}{\cos(A + B)\cos A} \qquad (2)$$

The compound-angle formulae:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

So

$$\sin A\cos(A + B) + \sin(A + B)\cos A = \sin(2A + B)$$

Substituting into (2):

$$\text{LHS} = \frac{\sin A\cos(A + B) + \sin(2A + B)}{\cos(A + B)\cos A} \qquad (3)$$

Product-to-sum identity:

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

So

$$\sin A\cos(A + B) = \frac{1}{2}\sin(2A + B) + \frac{1}{2}\sin(-B) = \frac{1}{2}\sin(2A + B) - \frac{1}{2}\sin B$$

Substituting into (3):

$$\text{LHS} = \frac{\frac{1}{2}\sin(2A + B) - \frac{1}{2}\sin B + \sin(2A + B)}{\cos(A + B)\cos A}$$

$$\text{LHS} = \frac{\frac{1}{2}(3\sin(2A + B) - \sin B\ )}{\cos(A + B)\cos A}$$

Given

$\sin B = 3\sin(2A + B)$

$3\sin(2A + B) - \sin B = 0$

Hence

$\text{LHS} = \text{RHS} = 0$