Question 1. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Show that if S is any subset of A with 7 elements, then there are 2 elements of S whose sum is 10.

Solution. There are 4 unordered pairs of numbers in A, whose sum equals 10:

$$\{1,9\},\{2,8\},\{3,7\},\{4,6\}.$$

The rest 2 numbers (5 and 10) do not have a corresponding complement to 10. Since 4+2=6<7, then if we choose arbitrary 7 numbers from A, by Pigeonhole principle we necessarily find at least one above pair among them.