Question #29146

Sum/difference trig problems please help urgent?
1.cos(345)
2.sin(405)
3.sin(10pi/24)
4.tan(-105)
answer has to be in rads please like 2pi/4

Solution.

1. It is known that $\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$.

Observe that $345^o=360^0-15^o=2\pi-\frac{\pi}{12}$ and $\cos(2\pi)=1,\sin(2\pi)=0$. Then

$$cos(345^o)=\cos\left(2\pi-\frac{\pi}{12}\right)=\cos(2\pi)\cos\left(\frac{\pi}{12}\right)+\sin(2\pi)\sin\left(\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{12}\right).$$

Taking into account that $\cos(\propto)=\sqrt{\frac{1+\cos(2\propto)}{2}}$ and $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, we obtain

$$cos(345^o)=\cos\left(\frac{\pi}{12}\right)=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}=\frac{\sqrt{2+\sqrt{3}}}{2}$$

Answer. $\frac{\sqrt{2+\sqrt{3}}}{2}$

2. Since $\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$, then

$$sin(405^o)=\sin(360^o+45^o)=\sin\left(2\pi+\frac{\pi}{4}\right)=\sin(2\pi)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\cos(2\pi)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

Answer. $\frac{\sqrt{2}}{2}$.

3. Observe that $\sin\left(\frac{10\pi}{24}\right)=\sin(\frac{12\pi-2\pi}{12})=sin\left(\frac{\pi}{2}-\frac{\pi}{12}\right)$. Since

$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$, we have

$$\sin\left(\frac{10\pi}{24}\right)=\sin\left(\frac{\pi}{2}-\frac{\pi}{12}\right)=\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{12}\right)-\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{2+\sqrt{3}}}{2}.$$

Answer. $\frac{\sqrt{2+\sqrt{3}}}{2}$.

4. It is known that $\tan(-\propto)=-\tan(\propto)$ and $\tan(\propto+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$. Then

$$tan(-105^0)=-tan(105^0)=-\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=-\frac{\tan\left(\frac{\pi}{3}\right)+\tan(\frac{\pi}{4})}{1-\tan\left(\frac{\pi}{3}\right)\tan(\frac{\pi}{4})}=-\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot1}=-\frac{\sqrt{3}+1}{1-\sqrt{3}}.$$

Answer. $-\frac{\sqrt{3}+1}{1-\sqrt{3}}$.