Task. What is the reason that the solution of $\sqrt{x} = -3$ is empty set?

Solution. The principal idea is that by definition $\sqrt{x}$ denotes non-negative number.

Let me describe this more precisely. It is known that for each $a > 0$ the equation

$$x^2 = a$$

has exactly two solutions. If z is one of them, then $-z$ is another one, since

$$(-z)^2 = z^2 = a.$$

Hence one of them is positive, and another one is negative.

Definition. The **positive** solution of the equation $x^2 = a$ is denoted by $\sqrt{a}$.

So the negative solution is $-\sqrt{a}$.

In particular, consider the equation

$$\sqrt{x} = -3.$$

The left hand side is, by definition, > 0, while the right hand side is < 0. Therefore this equation has no solutions.

On the ohter hand, the equation

$$\sqrt{x} = +3$$

has a unique solution

$$x = 9.$$