

A conical tank (with its tip down and the circular base parallel to and above the ground) is filling with water in such a way that the height of the water is increasing at a rate of  $0.1\text{cm/hr}$  at the instant that the height of the water level is  $10\text{cm}$ . If the tank has a radius of  $12\text{cm}$  and a height of  $30\text{cm}$ , find how fast the area corresponding to the top of the water level is increasing at this instant.

**Solution.**

1. It is given that the radius of the base of the tank  $R = 12\text{cm}$ , and the height of the tank  $H = 30\text{cm}$ . Denote as  $h(t)$  the level of the water at the moment  $t$  and  $r(t)$ -the radius of corresponding water surface. At the instant  $t_0$  the values  $h(t_0) = 10\text{cm}$  and  $\dot{h}(t_0) = 0.1\text{cm/hr}$ .

2. Considering the vertical cross section of the tank one may write

$$\frac{h(t)}{r(t)} = \frac{H}{R}.$$

Hence  $r(t) = \frac{R}{H}h(t)$  and the area of the water surface

$$S(t) = \pi r^2(t) = \frac{\pi R^2}{H^2} h^2(t).$$

Differentiating this expression we have

$$\dot{S}(t) = \frac{2\pi R^2}{H^2} h(t) \dot{h}(t),$$

Or at the moment  $t = t_0$

$$\dot{S}(t_0) = \frac{2\pi * 12^2}{30^2} * 10 * 0.1 \text{cm}^2/\text{hr} = \frac{72\pi}{225} \text{cm}^2/\text{hr}.$$

**Answer:**  $\frac{72\pi}{225} \text{cm}^2/\text{hr}$