

1. Graph one arch of the cycloid $x = t - \sin t$, $y = 1 - \cos t$.
2. Find the arc length of one arch of the above cycloid.

Using Maple

Solution.

1. Graph of one arch of the cycloid

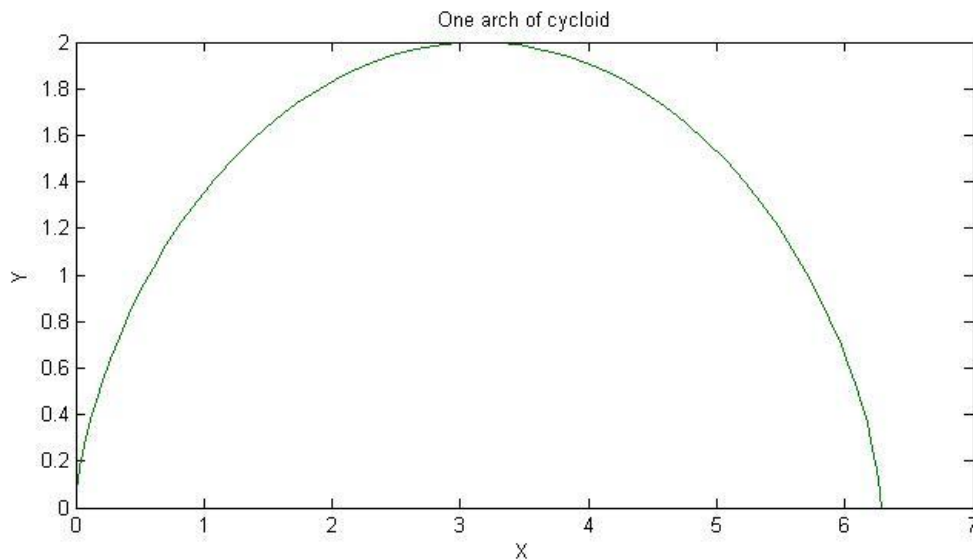
Here $t \in [0, 2\pi]$. So

$t = 0:0.1:2*\pi$;

$\gg x = t - \sin(t)$;

$\gg y = 1 - \cos(t)$;

$\gg \text{plot}(x, y)$



2. The arch length equals

$$l = \int_0^{2\pi} \sqrt{x'^2(t) + y'^2(t)} dt = \int_0^{2\pi} \sqrt{(1 - \cos t)^2 + \sin^2 t} dt = \int_0^{2\pi} \sqrt{2(1 - \cos t)} dt =$$

$$\int_0^{2\pi} \sqrt{4 \sin^2 \frac{t}{2}} dt = 2 \int_0^{2\pi} \sin \frac{t}{2} dt = -4 \cos \frac{t}{2} \Big|_0^{2\pi} = -4(\cos \pi - \cos 0) = 8.$$

Answer: 8.