Task. If the value of a $3 * 3$ determinant is 3, then the value of the determinant formed by its cofactors will be

a) 9

b) 3

c) 27

d) none of these

Solution. Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Recall that (i, j)-minor M_{ij} is the matrix obtained from A by removing i-th row and j-th column. Then the (i, j)-cofactor is defined as

$$C_{ij} = (-1)^{i+j} \det(M_{ij}).$$

For example, if $i = 1$ and $j = 3$, then

$$M_{13} = \begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

and

$$C_{13} = (-1)^{1+3} \det(M_{13}) = + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21}a_{32} - a_{31}a_{22}.$$

Let

$$C = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$

be the matrix consisting of cofactors and transposed.

Then it is known that

$$AC = \begin{pmatrix} \det(A) & 0 & 0 \\ 0 & \det(A) & 0 \\ 0 & 0 & \det(A) \end{pmatrix}$$

Hence

$$\det(A)\det(C) = \det(AC) = \det(A)^3.$$

Therefore

$$\det(C) = \det(A)^3 / \det(A) = \det(A)^2 = 3^2 = 9.$$

Answer. a) $\det(C) = 9$