Answer on Question #28951 – Math – Combinatorics | Number Theory

Question

How many permutations of letters of the word 'MADHUBANI' do not begin with M but end with I?

Solution

Let's find A=the number of permutations of letters in the word 'MADHUBANI', which end with I

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$ I

The last letter is I and we need to place 8 letters left: 1 M, 2 A, 1 D, 1 H, 1 U, 1 B, 1 N. It will be a multiset permutation. The number of multiset permutations is given by the multinomial coefficient

$\binom{n}{m_1, m_2, \ldots, m_k} = \frac{n!}{m_1! m_2! \ldots m_k!}$,

where $n = 8$;

$m_1 = 1, m_2 = 2, m_3 = 1, m_4 = 1, m_5 = 1, m_6 = 1, m_7 = 1$ are numbers of the letters M, A, D, H, U, B, N respectively.

So $A = \binom{n}{m_1, m_2, m_3, m_4, m_5, m_6, m_7} = \frac{8!}{1! \cdot 2! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} = \frac{8!}{2!}$.

Let's find B=the number of permutations of letters in the word 'MADHUBANI', which begin with M and end with I.

M $\_\ \_\ \_\ \_\ \_\ \_\ \_$ I

The first letter is M and the last letter is I and we need to place 7 letters left: 2 A, 1 D, 1 H, 1 U, 1 B, 1 N. It will be a multiset permutation. The number of multiset permutations is given by the multinomial coefficient

$\binom{n}{m_1, m_2, \ldots, m_k} = \frac{n!}{m_1! m_2! \ldots m_k!}$,

where $n = 7$;

$m_1 = 2, m_2 = 1, m_3 = 1, m_4 = 1, m_5 = 1, m_6 = 1$ are numbers of the letters A, D, H, U, B, N respectively.

So $B = \binom{n}{m_1, m_2, m_3, m_4, m_5, m_6} = \frac{7!}{2! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} = \frac{7!}{2!}$.

The number of permutations of letters in the word 'MADHUBANI', which do not begin with M but end with I will be

$N = A - B = \frac{8!}{2!} - \frac{7!}{2!} = \frac{8! - 7!}{2!} = \frac{8 \cdot 7! - 7!}{2!} = \frac{7!(8-1)}{2} = \frac{7 \cdot 7!}{2} = \frac{7 \cdot 5040}{2} = 17640$.

Answer: 17640.