

Task. How to find the derivative of a function $F(x) = \frac{\sin(2x)}{\cos x + 2}$.

Solution. We can use the formula

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

for the case $f(x) = \sin(2x)$ and $g(x) = \cos x + 2$.

Also notice that

$$f(h(x)) = f'(h(x)) \cdot h'(x).$$

Hence

$$f'(x) = (\sin(2x))' = \cos(2x) \cdot (2x)' = \cos(2x) \cdot 2 = 2 \cos(2x).$$

Thus

$$\begin{aligned} F'(x) &= \left(\frac{\sin(2x)}{\cos x + 2}\right)' = \frac{(\sin(2x))'(\cos x + 2) - \sin(2x) \cdot (\cos x + 2)'}{(\cos x + 2)^2} \\ &= \frac{2 \cos(2x)(\cos x + 2) - \sin(2x) \cdot (-\sin x)}{(\cos x + 2)^2} \\ &= \frac{2 \cos(2x) \cos x + 4 \cos(2x) + \sin(2x) \sin x}{(\cos x + 2)^2}. \end{aligned}$$