Task. The probability that a man will be alive in 25 years is 3/5 and probability for his wife in 25 year is 2/3. Find probability that:

a) both will be alive in 25 years;

b) only the wife will be alive in 25 years.

Solution. Let X be the event that "a man will be alive in 25 years", and Y be the event that "his wife will be alive in 25 years", so

$$P(X) = 3/5, \qquad P(Y) = 2/3.$$

Assume that these events are independent, so

$$P(X \text{ and } Y) = P(X) * P(Y).$$

a) We should find the probability that "both man and his wife will be alive in 25 years", i.e. $P(XY)$. Since they are independent, we obtain that

$$P(X \text{ and } Y) = P(X) * P(Y) = (3/5) * (2/3) = 2/5.$$

b) We should find the probability that "only the wife will be alive in 25 years", i.e. $P(\bar{X} \text{ and } Y)$, where $\bar{X}$ is the opposite event to X, that is

$$\bar{X} = \text{"a man will die before 25 years."}$$

Notice that

$$P(\bar{X}) = 1 - P(X) = 1 - 3/5 = 2/5.$$

Then

$$P(\bar{X} \text{ and } Y) = P(\bar{X}) \cdot P(Y) = (2/5) * (2/3) = 4/15.$$