Task. A and B are two students. Their chances of solving a problem correctly are 1/3 and 1/4. If the probability of their making a common error is 1/20 and they obtain the same answer then the probability of their answer to be correct is?

A. 1/12
B. 1/40
C. 13/120
D. 10/13

Solution. Consider the following events:
C_A = "A solved problem correctly"
C_B = "B solved problem correctly"
S = "A and B obtain the same answer"
C = "Their answer is correct"
E = "A and B made common error".
We will assume that the events C_A, C_B, and E independent, that is

$$P(C_A C_B E) = P(C_A) * P(C_B) * P(E),$$

$$P(C_A C_B) = P(C_A) * P(C_B), \qquad P(C_A E) = P(C_A) * P(E), \qquad P(C_B E) = P(C_B) * P(E).$$

We should find the conditional probability $P(C|S)$. By definition where SC is the intersection of events S and C:
SC = "A and B obtain correct answer"
By assumption we have that

$$P(C_A) = \frac{1}{3}, \qquad P(C_B) = \frac{1}{4}, \qquad P(E) = \frac{1}{20}.$$

We will express the probabilities $P(SC)$ and $P(S)$ via $P(C_A)$, $P(C_B)$, and $P(\overline{C_A}\,\overline{C_B})$.
Notice that events SC and $C_A C_B$ coincide:
SC = $C_A C_B$ = "A and B obtain correct answer",
so

$$P(SC) = P(C_A C_B) = P(C_A) * P(C_B) = \frac{1}{3} * \frac{1}{4} = \frac{1}{12}.$$

Now compute $P(S)$. Notice that S happens if either of the following two events hold:
$C_A C_B$ = "A and B made no errors'
$\overline{C_A C_B} E$ = "A and B made error and this error is the same".
These events are mutually exclusive, that is

$$P(S) = P(C_A C_B) + P(F).$$

Moreover,

$$P(F) = P(\overline{C_A C_B} E).$$

Since events C_A, C_B and E are independent, we obtain that

$$P(F) = P(\overline{C_A C_B} E) == P(\overline{C_A}) * P(\overline{C_B}) * P(E)$$
$$= \left(1 - \frac{1}{3}\right) * \left(1 - \frac{1}{4}\right) * \frac{1}{20} \qquad = \frac{2}{3} * \frac{3}{4} * \frac{1}{20} = \frac{1}{40},$$

so

$$P(S) = P(C_A C_B) + P(F) = \frac{1}{3} * \frac{1}{4} + \frac{1}{40} = \frac{1}{12} + \frac{1}{40} = \frac{13}{120}.$$

Therefore

$$P(C|S) = \frac{P(SC)}{P(S)} = \frac{P(C_A C_B)}{P(C_A C_B) + P(F)} = \frac{\frac{1}{12}}{\frac{13}{120}} = \frac{10}{13}.$$

Answer. D) $\frac{10}{13}$.