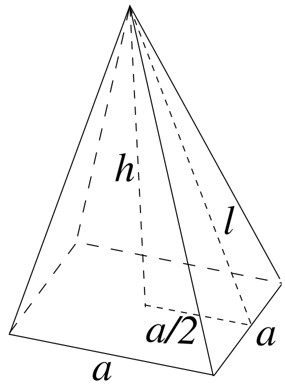
Task. Determine the surface of a pyramid with a base of 21 meters cubed and a height of 10 meters, and a pyramid with a base of 24 meters cubed and 8 meters in height.

Solution. The answer depends on the form of the base. However it is not clear from the assumptions what is the base: "21 meters cubed" is ambiguous.

So we assume that in both cases the base of the pyramid is a square and the vertex of pyramide is over the center of that square. Then the most probable formulation of the problem is the following: *find the area of the surface of the pyramid having volume 21 meters cubed and a height of 10 meters.* And similarly in the second case.

1) Notice that the surface of the pyramid consists of a base being a square and 4 faces being equal triangles, see Figure:



Let S_b be the area of the base, and S_f be the area of one of the faces. Then the area of all the surface of pyramid is

$$S = S_b + 4S_f.$$

We have that

$$V = 21m^3, \qquad h = 10m.$$

It is known that the volume of the pyramid is

$$V = \frac{1}{3}hS_b,$$

where S_b is the area of the base. Hence

$$S_b = \frac{3V}{h} = \frac{3 * 21}{10} = 6.3m^2.$$

It remains to find S_f.

The base of the pyramid is a square. Let a be its side. Then

$$S_b = a^2 = 6.3,$$

whence

$$a = \sqrt{6.3}m.$$

The face of the pyramid is a triangle and a is its base. So we should find the height l of this triangle. From figure we obtain that

$$l = \sqrt{h^2 + (a/2)^2} = \sqrt{10^2 + 6.3/4} = \sqrt{101.575}m.$$

Therefore the area of the face is

$$S_f = \frac{1}{2}la = \frac{1}{2}\sqrt{101.575}\sqrt{6.3} = \frac{1}{2}\sqrt{639.9225} = \sqrt{159.980625}m^2.$$

Hence the area of the pyramid is

$$S = S_b + 4S_f = 6.3 + 4\sqrt{159.980625} \approx 56.89m^2.$$

2) In the second case the arguments are literally the same. We have that

$$S_b = \frac{3V}{h} = \frac{3 * 24}{8} = 9,$$

$$a = \sqrt{S_b} = \sqrt{9} = 3,$$
$$l = \sqrt{h^2 + (a/2)^2} = \sqrt{8 + 9/4} = \sqrt{10.25}m.$$
$$S_f = \frac{1}{2}la = \frac{1}{2}\sqrt{10.25} \cdot 3 = \sqrt{23.0625}m^2,$$

and so the area of the pyramid is

$$S = S_b + 4S_f = 9 + 4\sqrt{23.0625} \approx 28.21m^2.$$