Question 1. *Show that, if x is a cluster point of $\{x^k\}$, and if $d(x, x^k) \geq d(x, x^{k+1})$, for all k, then x is the limit of the sequence.*

Solution. By definition of a cluster point for any $\varepsilon > 0$ there is $K \in \mathbb{N}$ such that $0 < d(x, x^K) < \varepsilon$. Then for arbitrary $k > K$ we have

$$d(x, x^k) \leq d(x, x^{k-1}) \leq \cdots \leq d(x, x^{K+1}) \leq d(x, x^K) < \varepsilon.$$

So, for every $\varepsilon > 0$ there is $K \in \mathbb{N}$ such that $d(x, x^k) < \varepsilon$ for all $k \geq K$. This exactly means that x is the limit of x^k. □