Question 1. *Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Show that if S is any subset of A with 7 elements, then there are 2 elements of S whose sum is 10.*

Solution. There are 4 unordered pairs of numbers in A, whose sum equals 10:

$$\{1, 9\}, \{2, 8\}, \{3, 7\}, \{4, 6\}.$$

The rest 2 numbers (5 and 10) do not have a corresponding complement to 10. Since $4 + 2 = 6 < 7$, then if we choose arbitrary 7 numbers from A, by Pigeonhole principle we necessarily find at least one above pair among them. □