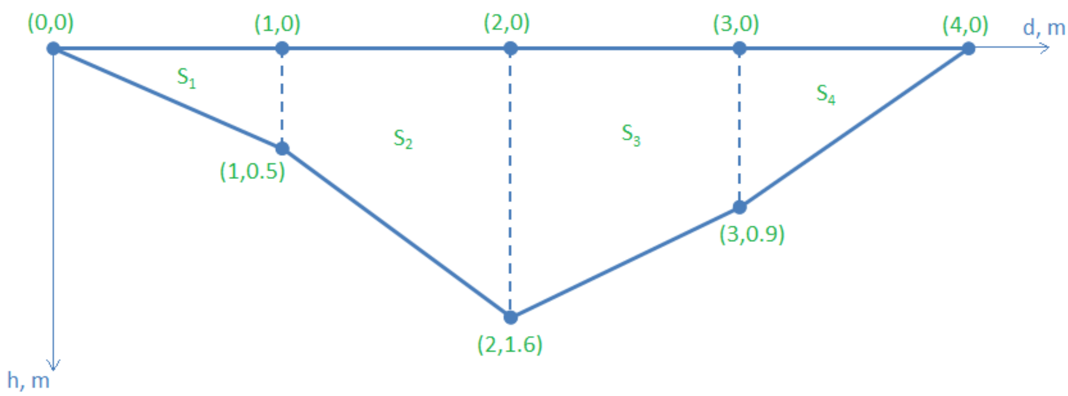
The table below gives the depth of water across a river measured at one metre intervals between banks. Distance (m) 0 1 2 3 4, water depth (m) 0 0.5 1.6 0.9 0. Use the trapezium rule to estimate the cross-sectional area of the river. A river hydrologist estimates that at the place where this cross sectional data was measured the average speed of water flow is 0.6m/s. Estimate the volume of water which passes this section of the river in one minute.

Solution.

d	0	1	2	3	4
h	0	0.5	1.6	0.9	0

The cross-sectional area of the water is shown on the graph:



Total area is: $S = S_1 + S_2 + S_3 + S_4$

Points of 1st triangle: $(0,0), (1,0), (1,0.5)$.

Area of 1st triangle: $S_1 = \frac{1}{2} \cdot (1-0) \cdot (0.5-0) = 0.25\ (m^2)$

2nd trapezium points: $(1,0), (2,0), (1,0.5), (2,1.6)$.

2nd trapezium area: $S_2 = \frac{(0.5-0)+(1.6-0)}{2} \cdot (2-1) = 1.05\ (m^2)$

3rd trapezium points: $(2,0), (3,0), (2,1.6), (3,0.9)$.

3rd trapezium area: $S_3 = \frac{(1.6-0)+(0.9-0)}{2} \cdot (3-2) = 1.25\ (m^2)$

Points of 4th triangle: $(3,0), (4,0), (3,0.9)$.

Area of 4th triangle: $S_4 = \frac{1}{2} \cdot (4-3) \cdot (0.9-0) = 0.45\ (m^2)$

So, *total area* is: $S = 0.25 + 1.05 + 1.25 + 0.45 = 3\ (m^2)$

The volume of water passes this section of the river in one minute is:

$$V = S \cdot v \cdot t = 3m^2 \cdot 0.6\frac{m}{s} \cdot 60s = 108\ (m^3)$$

Answer: $S = 3\ m^2,\ V = 108\ m^3$.