Task. Show that, if $\{s^k\}$ is bounded, then, for any element c in the metric space, there is a constant $t > 0$ with $d(c, s^k) \leq t$, for all k.

Proof. By definition, the set $\{s^k\}$ is bounded if there exists some $A > 0$ such that

$$d(s^k, s^l) \leq A$$

for all $k, l \geq 0$. Put

$$t = d(c, s^0) + A.$$

Then by triangle inequality for any k we have that

$$d(c, s^k) \leq d(c, s^0) + d(s^0, s^k) \leq d(c, s^0) + A = t.$$