

The table below gives the depth of water across a river measured at one metre intervals between banks. Distance (m) 0 1 2 3 4 Water depth (m) 0 0.5 1.6 0.9 0 Use the Trapezium rule to estimate the cross-sectional area of the river. A river hydrologist estimates that at the place where this cross sectional data was measured the average speed of water flow is 0.6m/s. Estimate the volume of water which passes this section of the river in one minute.

Solution

1st trapezium points: (0,0), (1,0), (0,0), (1,0.5).

$$1\text{st trapezium area: } S_1 = (1 - 0) * \min\{0,0.5\} + (1 - 0) * \frac{0.5-0}{2} = 0.25 \text{ m}^2$$

2nd trapezium points: (1,0), (2,0), (1,0.5), (2,1.6).

$$2\text{nd trapezium area: } S_2 = (2 - 1) * \min\{0.5,1.6\} + (2 - 1) * \frac{1.6-0.5}{2} = 1.05 \text{ m}^2$$

3rd trapezium points: (2,0), (3,0), (2,1.6), (3,0.9).

$$3\text{rd trapezium area: } S_3 = (3 - 2) * \min\{1.6,0.9\} + (3 - 2) * \frac{1.6-0.9}{2} = 1.25 \text{ m}^2$$

4th trapezium points: (3,0), (4,0), (3,0.9), (4,0).

$$4\text{th trapezium area: } S_4 = (4 - 3) * \min\{0.9,0\} + (4 - 3) * \frac{0.9-0}{2} = 0.45 \text{ m}^2$$

$$\text{Total area: } S = S_1 + S_2 + S_3 + S_4 = 0.25 + 1.05 + 1.25 + 0.45 = 3 \text{ m}^2$$

So, the volume of water passes this section of the river in one minute is approximately

$$V = 3\text{m}^2 * \left(\frac{0.6\text{m}}{\text{s}} * 60\text{s}\right) = 108 \text{ m}^3.$$

Answer: 3 m²; 108 m³.