

if $x = \sec\theta + \tan\theta$ then $x + \frac{1}{x} = ?$

$$\sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{So } x = \frac{1+\sin\theta}{\cos\theta}$$

$$\begin{aligned} \text{then } x + \frac{1}{x} &= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \\ &= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta. \end{aligned}$$