What is the number of permutations of the letters of the word HINDUSTAN such that neither the pattern "HIN" nor "DUS" nor "TAN" appears ?

Solution

Number of permutations of the letters of the word which has n letters is n!

If some of letters is doubled *Number of permutations* is $\frac{n!}{2}$, 2 letters are doubled

Number of permutations is $\frac{n!}{2*2}$ …

If we have fixed block than we work with it like as letter:

'HIN' 'D' 'U' 'S' 'T' 'A' 'N' – 7 letters, *Number of permutations* 7!

(a) Total number of permutations $= \frac{9!}{2}$, since N is repeated.

(b) Number of permutations in which HIN comes as a block = 7!

Number of permutations in which TAN comes as a block = 7!

Number of permutations in which DUS comes as a block $= \frac{7!}{2}$.

(c) This includes both HIN and TAN comes as blocks=5!
same is true for the other two pairs.

(d) Number of permutations in which hall three blocks come = 3!

∴ required number of permutations

$$=(a) - \{(b) - (c) + (d)\} = \frac{9!}{2} - \left\{7! + 7! + \frac{7!}{2} + 3(5!) + 3!\right\} =$$

$$= \frac{362880}{2} - \left\{5040 + 5040 + \frac{5040}{2} - 3*120 + 6\right\} = 169194$$

Answer: 169194.