Task. Prove

$$\cot(a-b) - \tan(a-b) = \frac{2\sin 2a}{\sin 2a + \sin 2b}$$

Proof. This identity is not correct. For instance let $a = \frac{\pi}{3}$ and $b = \frac{\pi}{6}$. Then

$$a - b = \frac{\pi}{6},$$

$$\cot(a-b) = \sqrt{3}, \qquad \tan(a-b) = \frac{1}{\sqrt{3}},$$

$$\sin 2a = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}, \qquad \sin 2b = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2},$$

so

$$\cot(a-b) - \tan(a-b) = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}},$$

while

$$\frac{2\sin 2a}{\sin 2a + \sin 2b} = \frac{2\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{\sqrt{3}} = 1 \neq \frac{2}{\sqrt{3}}.$$

Notice that the left hand side can be simplified as follows:

$$\begin{aligned}\cot(a-b) - \tan(a-b) &= \frac{\cos(a-b)}{\sin(a-b)} - \frac{\sin(a-b)}{\cos(a-b)} = \frac{\cos^2(a-b) - \sin^2(a-b)}{\sin(a-b)\cos(a-b)} = \\ &= \frac{2\cos 2(a-b)}{2\sin(a-b)\cos(a-b)} = \frac{2\cos 2(a-b)}{2\sin(a-b)\cos(a-b)} = \\ &= \frac{2\cos 2(a-b)}{\sin 2(a-b)} = 2\cot 2(a-b).\end{aligned}$$