Consider a parabola y=x^(2)-4x+8.

(a) Write down two straight lines that touch the parabola at only one point.

(b) Find the corresponding point of contact in (a).

a)

$y = x^2 - 4x + 8$

Slope of tangent at any point is:

$\frac{dy}{dx} = 2x - 4$

Suppose, line that touches the parabola at the only one point x_0 is $y = ax + b$. Then

$a = \frac{dy}{dx}|_{x=x_0}$ and $ax_0 + b = x_0^2 - 4x_0 + 8$.

So

$a = 2x_0 - 4$

$ax_0 + b = x_0^2 - 4x_0 + 8$ => $b = x_0^2 - 4x_0 + 8 - (2x_0 - 4)x_0 = -x_0^2 + 8$

For example $x_0 = 2$

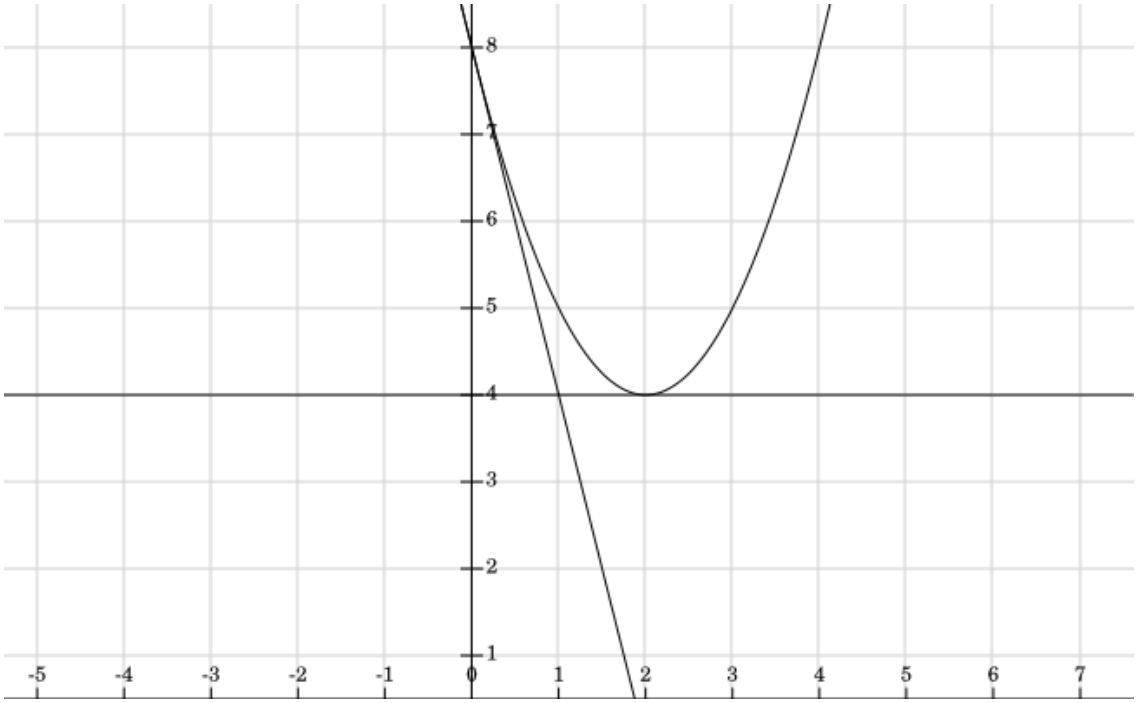
$a = 0, \quad b = 4$

$\boldsymbol{y = 4}$

And $x_0 = 0$:

$a = -4 \quad b = 8$

$\boldsymbol{y = -4x + 8}$



Answer: $y = -4x + 8, y = 4$

b)

Point of contact:

$\begin{cases} y = 4 \\ y = -4x + 8 \end{cases}$ => $4 = -4x + 8$ => $x = 1$

Answer $x = 1, y = 4$