

By using divergence theory compute the surface integral of  $\int_T \vec{F} \cdot \vec{n} ds$  where  $T$  is the unit sphere  $x^2+y^2+z^2=1$ . and the vector field  $\vec{F}$  is  $(y, z, x)$ .

I found the  $\text{div}(\vec{F})=0$  so the surface integral is zero? Is this correct? Could someone please confirm? Thank you!

**Solution.**

1. Ostrogradsky-Gauss theorem states that the flow of the vector field  $\vec{F}$  through the closed surface  $T$  which limits the volume  $V$  equals the integral of divergence of the vector field on this volume, namely,

$$\iint_T (\vec{F} \cdot \vec{n}) ds = \iiint_V \text{div} \vec{F} dv.$$

2. In the given case  $V : x^2 + y^2 + z^2 \leq 1, T : x^2 + y^2 + z^2 = 1, \vec{F} = (y, z, x)$ .

3. As  $\text{div} \vec{F} = 0$  then  $\iint_T (\vec{F} \cdot \vec{n}) ds = 0$ .