Find the volume of the solid obtained by rotating the region bounded by $x = 4y^2 - y^3$, $x = 0$ about $y = 0$

Solution.

1.Finding the maximal value of x we get that the given figure is bounded by upper line $x = 4y^2 - y^3$ for $y \in [\frac{8}{3}, 4]$ and lower line $x = 4y^2 - y^3$ for $y \in [0, \frac{8}{3}]$.

2.Write the equation in a parametric form

$$\begin{cases} y = 4\sin t \\ x = 64\sin^2 t(1-\sin t). \end{cases} \text{ where } t \in [0, pi/2]. \qquad (1)$$

And note that for $y = \frac{8}{3}$ the value of $t = \arcsin\frac{2}{3}$.

3.The desired volume = the volume formed by rotation of the upper line $t \in [pi/2, \arcsin\frac{2}{3}]$ - the volume formed by the lower line $t \in [0, \arcsin\frac{2}{3}]$.

That is

$$V = pi \int_{pi/2}^{\arcsin\frac{2}{3}} y^2(t)x'(t)dt - pi \int_{0}^{arccin\frac{2}{3}} y^2(t)x'(t)dt ,$$

or

$$V = -pi \int_{0}^{pi/2} y^2(t)x'(t)dt . \qquad (2)$$

4.Substituting (1) into (2) and evaluating the integral we 'll get

$$V = \frac{512}{5} pi.$$

Answer: $V = \frac{512}{5} pi.$