Answer on Question # 28347 – Math – Calculus

integrate 1/w(4-2w)^1/2

Solution.

Take the integral:

$$\int \frac{\sqrt{4-2w}}{w}\, dw$$

For the integrand $\frac{\sqrt{4-2w}}{w}$, substitute $u = \sqrt{4-2w}$ and $du = -\frac{1}{\sqrt{4-2w}}\, dw$:

$$= -\int \frac{2u^2}{4-u^2}\, du$$

Factor out constants:

$$= -2\int \frac{u^2}{4-u^2}\, du$$

For the integrand $\frac{u^2}{4-u^2}$, cancel common terms in the numerator and denominator:

$$= -2\int -\frac{u^2}{u^2-4}\, du$$

Factor out constants:

$$= 2\int \frac{u^2}{u^2-4}\, du$$

For the integrand $\frac{u^2}{u^2-4}$, do long division:

$$= 2\int \left(-\frac{1}{u+2} + \frac{1}{u-2} + 1\right) du$$

Integrate the sum term by term and factor out constants:

$$= 2\int 1\, du + 2\int \frac{1}{u-2}\, du - 2\int \frac{1}{u+2}\, du$$

For the integrand $\frac{1}{u+2}$, substitute $s = u + 2$ and $ds = du$:

$$= -2 \int \frac{1}{s}\, ds + 2 \int 1\, du + 2 \int \frac{1}{u-2}\, du$$

For the integrand $\frac{1}{u-2}$, substitute $p = u - 2$ and $dp = du$:

$$= 2 \int \frac{1}{p}\, dp - 2 \int \frac{1}{s}\, ds + 2 \int 1\, du$$

The integral of $\frac{1}{p}$ is $\log(p)$:

$$= 2 \log(p) - 2 \int \frac{1}{s}\, ds + 2 \int 1\, du$$

The integral of $\frac{1}{s}$ is $\log(s)$:

$$= 2 \log(p) - 2 \log(s) + 2 \int 1\, du$$

The integral of 1 is u:

$$= 2 \log(p) - 2 \log(s) + 2u + \text{constant}$$

Substitute back for $p = u - 2$:

$$= -2 \log(s) + 2u + 2 \log(u-2) + \text{constant}$$

Substitute back for $s = u + 2$:

$$= 2u + 2 \log(u-2) - 2 \log(u+2) + \text{constant}$$

Substitute back for $u = \sqrt{4 - 2w}$:

$$= 2\sqrt{4-2w} + 2 \log\left(\sqrt{4-2w} - 2\right) - 2 \log\left(\sqrt{4-2w} + 2\right) + \text{constant}$$

Factor the answer a different way:

$$= 2\left(\sqrt{4-2w} + \log\left(\sqrt{4-2w} - 2\right) - \log\left(\sqrt{4-2w} + 2\right)\right) + \text{constant}$$

An alternative form of the integral is:

$$= 2\left(\sqrt{4-2w} + \log\left(\frac{\sqrt{4-2w} - 2}{\sqrt{4-2w} + 2}\right)\right) + \text{constant}$$

Which is equivalent for restricted w values to:

Answer:

$$= 2\sqrt{4-2w} + 2 \log\left(2 - \sqrt{4-2w}\right) - 2 \log\left(\sqrt{4-2w} + 2\right) + \text{constant}$$