A registered golden retriever has a litter of 11 puppies. Assume that the probability of a puppy being male is .5

1. Because the owner of the dof can expect to get more money for a male puppy, what is the most likely number of males in the litter?

Let N be the number of males in the litter.

$$P(N = k) = \binom{11}{k} \cdot \frac{1}{2^{11}}$$

$$\max_k P(N = k) = \frac{\max_k \binom{11}{k}}{2^{11}} = \frac{\binom{11}{6}}{2^{11}} = \frac{\binom{11}{5}}{2^{11}} = P(N = 6) = P(N = 5)$$

So the most likely number of males is 6 or 5.

2. What is the probability at least 7 of the puppies will be male?

$$P(N \geq 7) = \frac{\binom{11}{7} + \binom{11}{8} + \binom{11}{9} + \binom{11}{10} + \binom{11}{11}}{2^{11}} =$$

$$= \frac{\frac{8 \cdot 9 \cdot 10 \cdot 11}{4!} + \frac{9 \cdot 10 \cdot 11}{3!} + \frac{10 \cdot 11}{2!} + \frac{11}{1!} + 1}{2^{11}} = \frac{11(30 + 15 + 5 + 1) + 1}{2^{11}} =$$

$$= \frac{11 \cdot 51 + 1}{2^{11}} = \frac{562}{2^{11}} = \frac{281}{1024}$$