Task. What is the 90% confidence interval for a population of 482, means of 164 standard deviation of 33.

Solution. Thus we measure some numerical characteristic X of these peoples and this characteristic has mean $\mu = 164$ and standard deviation $\sigma = 33$. We take a sample of $n = 482$ people. Then the sample mean has normal distribution $N(\mu, \sigma/\sqrt{n})$.

Therefore the 90% interval has the form

$$(\mu - \delta, \ \mu + \delta),$$

where δ is choosen so that

$$P(|X - \mu| < \delta) = 0.9.$$

Consider another random variable

$$Z = \frac{X - \mu}{\sigma/\sqrt{n}}.$$

Then Z has standard normal distribution $N(0, 1)$ and the values of its distribution function

$$F(t) = P(Z \leq t)$$

can usually be found in any book in Probability theory.

Notice that

$$X - \mu = Z\sigma/\sqrt{n},$$

and so

$$P(|X - \mu| < \delta) = P(|Z\sigma/\sqrt{n}| \leq \delta) = P(|Z| \leq \delta\sqrt{n}/\sigma) = 0.9$$

Denote

$$a = -\delta\sqrt{n}/\sigma.$$

Since the normal distribution is symmetrical,

$$P(|Z| < a) = 1 - 2F(a),$$

whence

$$P(|X - \mu| < \delta) = 1 - 2F(a) = 0.9,$$

whence

$$F(a) = (1 - 0.9)/2 = 0.05.$$

Then from tables of values of F we obtain that

$$a = 1.645.$$

Thus

$$a = \delta\sqrt{n}/\sigma = 1.645,$$

whence

$$\delta = 1.645 \cdot \frac{\sigma}{\sqrt{n}} = \frac{1.645 * 33}{\sqrt{482}} = 2.4726.$$

Therefore 90% confidence interval is the following:

$$(164 - 2.4726, 164 + 2.4726)$$

$$(161.5274, 166.4726).$$