

1. Suppose 51% of the banks in Switzerland are private organization. If a sample of 430 banks is selected, what is the probability that the sample proportion of private banks will be less than 52%.

**Solution.**

Let's  $X_i, i = 1 \dots 430$  is random variables such that:

$$X_i = \begin{cases} 1, & \text{bank is private} \\ 0, & \text{bank is not private} \end{cases} . P(X_i = 1) = 0.51 \text{ and } P(X_i = 0) = 0.49.$$

Let  $S = \sum_{i=1}^{430} X_i$ . Now calculate the mean and the variance of  $S$ .

$$E[S] = 430 * P(X_i = 1) = 219.3$$

$$Var[S] = 430 * P(X_i = 1) * P(X_i = 0) = 107.45$$

Let variable  $Z = \frac{S - E[S]}{\sqrt{Var[S]}}$ . It is easy to see that  $E[Z] = 0, Var[Z] = 1$ .

Using Central Limit theorem we can say that  $Z \sim N(0,1)$ .

$$P(S < 0.52 * 430) = P(S < 223.6) = P(Z * \sqrt{Var[S]} + E[S] < 223.6), \text{ then}$$

$$P(Z * 10.36 + 219.3 < 223.6) = P(Z * 10.36 < 4.3) = P(Z < 0.41) = \Phi(0.41)$$

$P(S < 0.52 * 430) = \Phi(0.41) = 0.6591$ . Here we used the table of the normal distribution.

**Answer: 0.6591.**