

A bag contains three red, four white and five black balls. If three balls are taken without replacement, what is the probability that:

(i) they are all the same colour

Probability what they are all the same colour equals:

$$P = P(r) + P(w) + P(b)$$

$P(r), P(w), P(b)$ - probability what they are all red, white or black colour

$$P(r) = P(r_1) * P(r_2) * P(r_3)$$

$P(r_1), P(r_2), P(r_3)$ - probability what 1st, 2nd and 3rd taken ball is red.

$$P(r_1) = \frac{3}{3+4+5} = \frac{3}{12} = \frac{1}{4} - \text{we have 3 red balls and total 12.}$$

$$P(r_2) = \frac{2}{2+4+5} = \frac{2}{11} - \text{we have 2 red balls and total 11.}$$

$$P(r_3) = \frac{1}{1+4+5} = \frac{1}{10} - \text{we have 1 red ball and total 10.}$$

$$P(r) = P(r_1) * P(r_2) * P(r_3) = \frac{1}{4} \frac{2}{11} \frac{1}{10} = \frac{1}{220}$$

For $P(w), P(b)$ all almost the same:

$$P(w) = \frac{4}{12} \frac{3}{11} \frac{2}{10} = \frac{1}{3} \frac{3}{11} \frac{1}{5} = \frac{1}{55}$$

$$P(b) = \frac{5}{12} \frac{4}{11} \frac{3}{10} = \frac{5}{12} \frac{4}{11} \frac{3}{10} = \frac{1}{3} \frac{1}{11} \frac{3}{2} = \frac{1}{22}$$

$$P = P(r) + P(w) + P(b) = \frac{1}{220} + \frac{1}{55} + \frac{1}{22} = \frac{3}{44}$$

Answer: $\frac{3}{44}$

(ii) there are no black balls

Probability what there are no black balls:

$$P(b) = P(b_1) * P(b_2) * P(b_3)$$

$P(b_1), P(b_2), P(b_3)$ - probability what 1st, 2nd and 3rd taken ball is no black.

$$P(b_1) = \frac{7}{12} - \text{we have 7 no black balls and total 12}$$

$$P(b_2) = \frac{7}{11} - \text{we have 7 no black balls and total 11}$$

$$P(b_3) = \frac{7}{10} - \text{we have 7 no black balls and total 10}$$

$$P(b) = P(b_1) * P(b_2) * P(b_3) = \frac{7}{12} \frac{7}{11} \frac{7}{10} = \frac{343}{1320}$$

Answer: $\frac{343}{1320}$

(iii) there are 2 black balls

We can take 2 black balls:

$$\text{black, black, no black: } P(1) = \frac{5}{12} \frac{4}{11} \frac{7}{10}$$

$$\text{black, no black, black: } P(2) = \frac{5}{12} \frac{7}{11} \frac{4}{10}$$

$$\text{no black, black, black: } P(3) = \frac{7}{12} \frac{5}{11} \frac{4}{10}$$

$$P = P(1) + P(2) + P(3) = 3 * \frac{7}{12} \frac{5}{11} \frac{4}{10} = \frac{7}{22}$$

Answer: $\frac{7}{22}$

(iv) there is one ball of each colour

We have $3!$ combinations like:

brw, bwr, wbr, wrb, rbw, rwb (for example brw means black, red, white)

So total probability:

$$P = 3! * P(brw) = 6 * \frac{5}{12} \frac{4}{11} \frac{3}{10} = \frac{3}{11}$$

Answer: $\frac{3}{11}$