

### Question #28030

find to the nearest degree all values of theta in the interval  $0 \leq \theta < 360$  that satisfy  $3 \cos^2 \theta + \sin \theta - 1 = 0$

**Solution.** Using the trigonometric formulas  $\cos^2 \alpha + \sin^2 \alpha = 1$  and  $\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$ , we can simplify:

$$3 \cos(2\theta) + \sin \theta - 1 = 0$$

$$6 \sin^2 \theta - \sin \theta - 2 = 0$$

In other words we shall solve the quadratic equation.

$$D = 1 + 4 \cdot 6 \cdot 2 = 49.$$

Thus,

$$\sin \theta = \frac{1+7}{12} = \frac{2}{3} \text{ and } \sin \theta = \frac{1-7}{12} = -\frac{1}{2}.$$

$$\theta = (-1)^k \arcsin(\theta) + 180^\circ k, k \in \mathbb{Z}.$$

Since  $0 \leq \theta < 360$ , then  $\theta = 42^\circ, 138^\circ, 210^\circ, 330^\circ$ .

**Answer.**  $42^\circ, 138^\circ, 210^\circ, 330^\circ$ .