1. What is the general solution of $e^{\frac{1}{\sqrt{2}}}\left(e^{\sin x}+e^{\cos x}\right)=2$?

Solution.

$e^{\frac{1}{\sqrt{2}}}\left(e^{\sin x}+e^{\cos x}\right)=2$ equal to $\left(1-e^{\frac{1}{\sqrt{2}}+\sin x}\right)+\left(1-e^{\frac{1}{\sqrt{2}}+\cos x}\right)=0$

It is easily to see that $\sin x+\frac{1}{\sqrt{2}}=0$ and $\cos x+\frac{1}{\sqrt{2}}=0$ one of the solutions.

And this is equal to $x_0=\frac{5\pi}{4}+2\pi k, k\in Z$. We have obtain that x_0 is the minimum of the function in the left side of the equation, because $e^{\sin x}+e^{\cos x}$ is 2π-periodic function, and If it were not so we would have 2 solutions on the every interval $(2\pi k, 2\pi(k+1)]$ instead of one. So let's find the minimum of the left side:

$min=e^{\frac{1}{\sqrt{2}}}\left(e^{\sin\left(\frac{5\pi}{4}+2\pi k\right)}+e^{\cos\left(\frac{5\pi}{4}+2\pi k\right)}\right)$, it follows that $min=2$. It is easy to see that this is not exactly maximum, because if $x=\frac{\pi}{2}$ we have $e^{\frac{1}{\sqrt{2}}}\left(e^{\sin\frac{\pi}{2}}+e^{\cos\frac{\pi}{2}}\right)=e^{1+\frac{1}{\sqrt{2}}}>min=2$. And now we have got perfect situation, when left side of the equation more than 2, and right side of the equation is equal 2. We conclude that there is no any other solution instead of x_0.

Answer: $\frac{5\pi}{4}+2\pi k, k\in Z$.